

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

**Computation Tree Logic**

    syntax and semantics of CTL

    expressiveness of CTL and LTL



    CTL model checking

    fairness, counterexamples/witnesses

    CTL<sup>+</sup> and CTL\*

Equivalences and Abstraction

# Equivalence of CTL and LTL formulas

COMPARISON4.2-1

Let  $\phi$  be a **CTL** formula and  $\varphi$  an **LTL** formula.

Let  $\phi$  be a **CTL** formula and  $\varphi$  an **LTL** formula.

$\phi \equiv \varphi$  iff for all transition systems  $\mathcal{T}$  and all states  $s$  in  $\mathcal{T}$ :

$$s \models_{\text{CTL}} \phi \iff s \models_{\text{LTL}} \varphi$$

# Equivalence of CTL and LTL formulas

Let  $\Phi$  be a **CTL** formula and  $\varphi$  an **LTL** formula.

$\Phi \equiv \varphi$  iff for all transition systems  $\mathcal{T}$  and all states  $s$  in  $\mathcal{T}$ :

$$s \models_{\text{CTL}} \Phi \iff s \models_{\text{LTL}} \varphi$$

e.g.,

CTL formula  $\Phi$

LTL formula  $\varphi$

$a$

$a$

$\forall \bigcirc a$

$\bigcirc a$

$\forall (a \cup b)$

$a \cup b$

$a, b \in AP$

# More examples

CTL formula $\Phi$	LTL formula $\varphi$
$a$	$a$
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$

# More examples

CTL formula $\Phi$	LTL formula $\varphi$
$a$	$a$
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$

# More examples

COMPARISON4.2-1A

CTL formula $\Phi$	LTL formula $\varphi$
$a$	$a$
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$
$\forall \square \diamond a$	$\square \diamond a$



# More examples

CTL formula $\Phi$	LTL formula $\varphi$
$a$	$a$
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{W} b)$	$a \text{W} b$
$\forall \square \forall \diamond a$	$\square \diamond a$

infinately often  $a$

# More examples

CTL formula $\Phi$	LTL formula $\varphi$
$a$	$a$
$\forall \bigcirc a$	$\bigcirc a$
$\forall (a \cup b)$	$a \cup b$
$\forall \square a$	$\square a$
$\forall \diamond a$	$\diamond a$
$\forall (a \text{ W } b)$	$a \text{ W } b$
$\forall \square \forall \diamond a$	$\square \diamond a$

infinately often  $a$

but:  $\forall \diamond \forall \square a \not\equiv \diamond \square a$

# The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$  iff on each path  $\pi$  from  $s$   
there is a state  $t$  with  $t \models \forall \square a$

# The CTL formula $\forall\Diamond\forall\Box a$

COMPARISON4.2-2

$s \models \forall\Diamond\forall\Box a$  iff on each path  $\pi$  from  $s$   
there is a state  $t$  with  $t \models \forall\Box a$

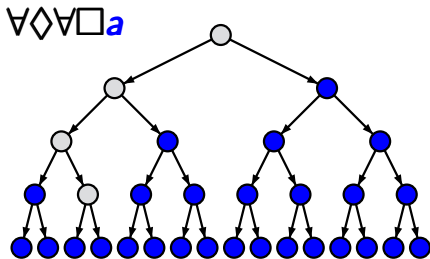
i.e., all states in the computation tree of  $t$  fulfill  $a$

# The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$  iff on each path  $\pi$  from  $s$   
there is a state  $t$  with  $t \models \forall \square a$

i.e., all states in the computation tree of  $t$  fulfill  $a$





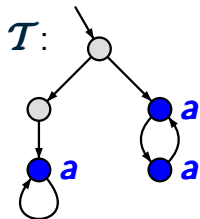
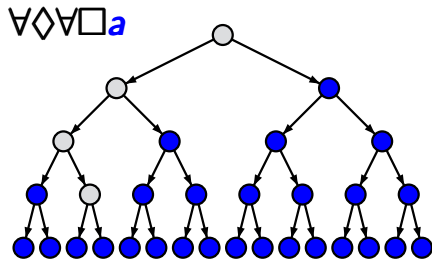
# The CTL formula $\forall \diamond \forall \square a$

COMPARISON4.2-2

$s \models \forall \diamond \forall \square a$  iff on each path  $\pi$  from  $s$   
there is a state  $t$  with  $t \models \forall \square a$



i.e., all states in the computation tree of  $t$  fulfill  $a$



$\mathcal{T} \models \forall \diamond \forall \square a$



$\exists \square a \neq \forall \diamond \forall \square a$

COMPARISON 4.2-3

$$\Diamond \Box a \neq \forall \Diamond \forall \Box a$$

To prove that

$$\forall \Diamond \forall \Box a \neq \Diamond \Box a$$

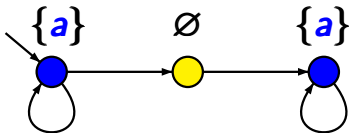
we provide an example for a TS  $\mathcal{T}$  s.t.

$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond \forall \Box a$$

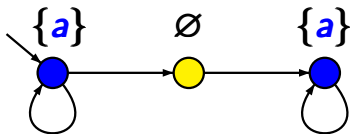
$$\diamond \square a \not\equiv \forall \diamond \forall \square a$$

transition system  $\mathcal{T}$



$$\Diamond \Box a \neq \forall \Diamond \forall \Box a$$

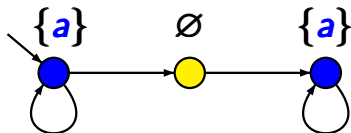
transition system  $\mathcal{T}$



$$\mathcal{T} \models_{\text{LTL}} \Diamond \Box a$$

$$\diamond \square a \neq \forall \diamond \forall \square a$$

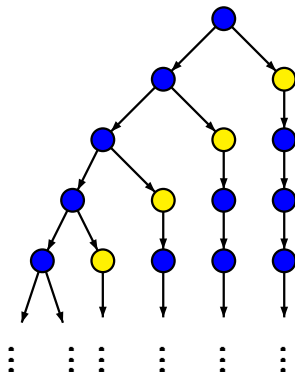
transition system  $\mathcal{T}$



$$\mathcal{T} \models_{\text{LTL}} \diamond \square a$$

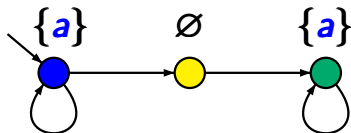
$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond \forall \square a$$

computation tree



$$\diamond \square a \neq \forall \diamond \forall \square a$$

transition system  $\mathcal{T}$

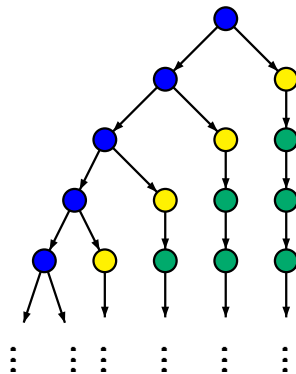


$$\mathcal{T} \models_{\text{LTL}} \diamond \square a$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond \forall \square a$$

$$\text{Sat}(\forall \square a) = \{\bullet\}$$

computation tree



# From CTL to LTL, if possible

COMPARISON4.2-4

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or ...

*without proof*



For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$   
by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \diamond \forall \square a$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \diamond \forall \square a$$

↓

$$\varphi = \diamond \square a$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \diamond \forall \square a$$

↓

$$\varphi = \diamond \square a \not\equiv \Phi$$

For each **CTL** formula  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL** formula obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \diamond \forall \square a$$

↓

$$\varphi = \diamond \square a \not\equiv \Phi$$

*hence:* there is no LTL formula equivalent to  $\Phi$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \square \forall \diamond a$$

For each **CTL** formula  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL** formula obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \square \forall \diamond a$$

↓

$$\varphi = \square \diamond a$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \square \forall \diamond a$$

↓

$$\varphi = \square \diamond a \equiv \Phi$$

“infinitely often  $a$ ”



For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \Diamond (a \wedge \forall \bigcirc a)$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \diamond (a \wedge \forall \bigcirc a)$$

↓

$$\varphi = \diamond (a \wedge \bigcirc a)$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\begin{aligned}\Phi &= \forall \Diamond (a \wedge \forall \bigcirc a) \\ \downarrow \\ \varphi &= \Diamond (a \wedge \bigcirc a) \neq \Phi\end{aligned}$$

For each **CTL formula**  $\Phi$  the following holds:

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$

where  $\varphi$  is the **LTL formula** obtained from  $\Phi$  by removing of all path quantifiers  $\exists$  and  $\forall$

*without proof*

$$\Phi = \forall \Diamond (a \wedge \forall \bigcirc a)$$

↓

$$\varphi = \Diamond (a \wedge \bigcirc a) \not\equiv \Phi$$

*hence:* there is no LTL formula equivalent to  $\Phi$

$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$

COMPARISON4.2-4A

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$

To prove that

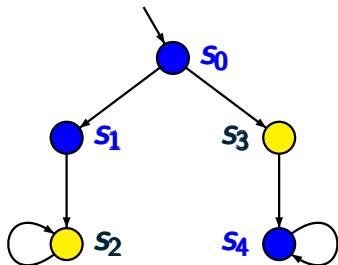
$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$

we provide an example for a TS  $\mathcal{T}$  s.t.

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

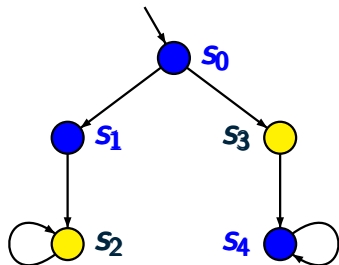
$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow circle} = \emptyset$$

$$\text{Blue circle} = \{a\}$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



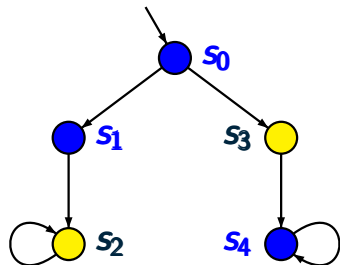
$$\text{Yellow circle} = \emptyset$$

$$\text{Blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$



$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



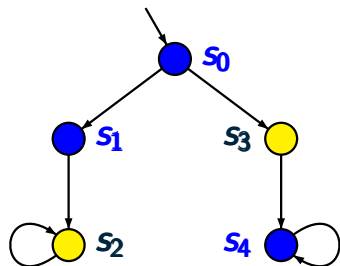
$$\text{yellow circle} = \emptyset$$

$$\text{blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



$$\text{yellow circle} = \emptyset$$

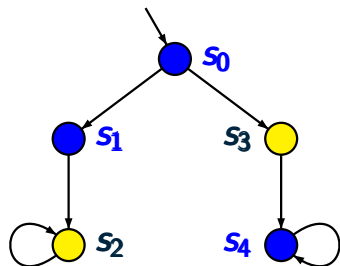
$$\text{blue circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

$$\diamond(a \wedge \bigcirc a) \not\equiv \forall \diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow Circle} = \emptyset$$

$$\text{Blue Circle} = \{a\}$$

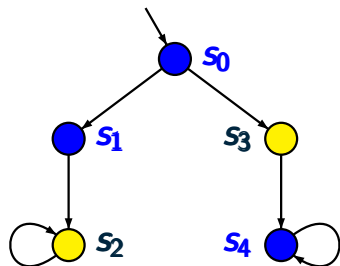
$$\mathcal{T} \models_{\text{LTL}} \diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \diamond(a \wedge \forall \bigcirc a)$$

$$\text{Sat}(a \wedge \forall \bigcirc a) = \{s_4\}$$

$$\Diamond(a \wedge \bigcirc a) \not\equiv \forall \Diamond(a \wedge \forall \bigcirc a)$$



$$\text{Yellow Circle} = \emptyset$$

$$\text{Blue Circle} = \{a\}$$

$$\mathcal{T} \models_{\text{LTL}} \Diamond(a \wedge \bigcirc a)$$

$$\begin{aligned} \text{trace}(s_0 s_1 s_2^\omega) &= \{a\} \{a\} \emptyset^\omega \\ \text{trace}(s_0 s_3 s_4^\omega) &= \{a\} \emptyset \{a\}^\omega \end{aligned}$$

$$\mathcal{T} \not\models_{\text{CTL}} \forall \Diamond(a \wedge \forall \bigcirc a)$$

$$\begin{aligned} \text{Sat}(a \wedge \forall \bigcirc a) &= \{s_4\} \\ s_0 s_1 s_2^\omega &\not\models_{\text{CTL}} \Diamond(a \wedge \forall \bigcirc a) \end{aligned}$$

The expressive powers of **LTL** and **CTL** are incomparable

The expressive powers of **LTL** and **CTL** are incomparable

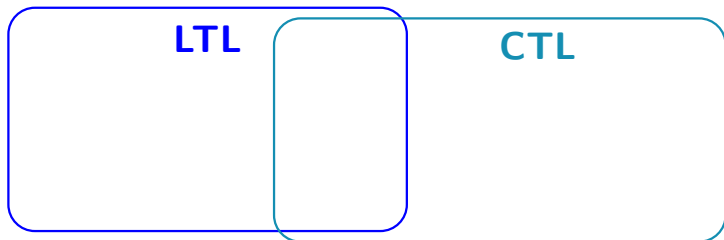
- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula

The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula

The expressive powers of **LTL** and **CTL** are incomparable

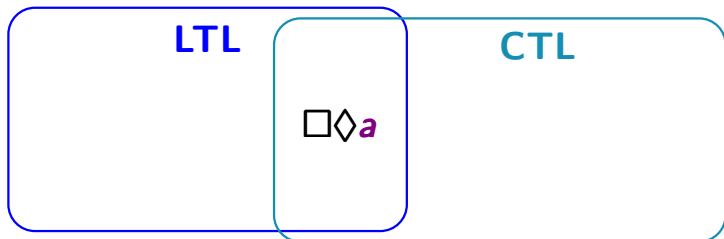
- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula





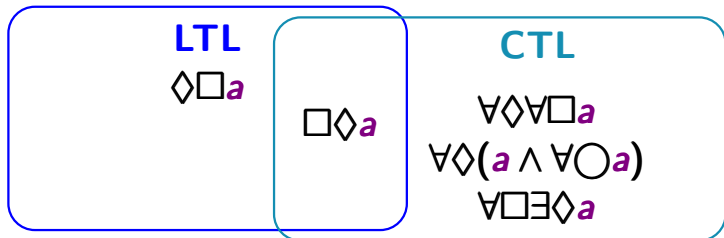
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula



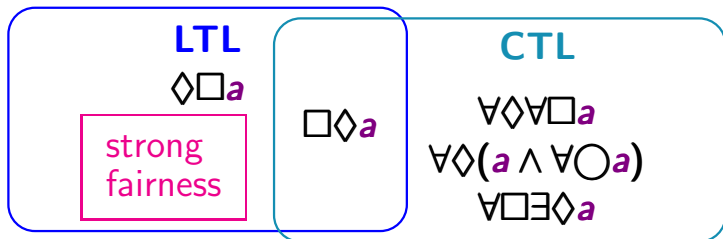
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula



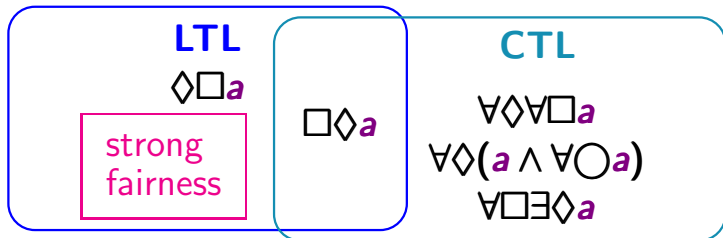
The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula



The expressive powers of **LTL** and **CTL** are incomparable

- The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent LTL formula
- The **LTL** formula  $\Diamond\Box a$  has no equivalent CTL formula



The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \square a$$

$$\forall \square \exists \Diamond a$$

have no equivalent **LTL** formula

The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \Box a$$

$$\forall \Box \exists \Diamond a$$

have no equivalent **LTL** formula

*Proof* uses the fact that for each **CTL** formula  $\Phi$ :

- either there is **no** equivalent **LTL** formula
- or  $\Phi \equiv \varphi$  where  $\varphi$  is the **LTL** formula obtained from  $\Phi$  by removing of all path quantifiers

The CTL formulas

$\forall \Diamond (a \wedge \forall \bigcirc a)$  ← already considered

$\forall \Diamond \forall \square a$  ← already considered

$\forall \square \exists \Diamond a$

have no equivalent LTL formula

*Proof* uses the fact that for each CTL formula  $\Phi$ :

- either there is **no** equivalent LTL formula
- or  $\Phi \equiv \varphi$  where  $\varphi$  is the LTL formula obtained from  $\Phi$  by removing of all path quantifiers

The **CTL** formulas

$$\forall \Diamond (a \wedge \forall \bigcirc a)$$

$$\forall \Diamond \forall \Box a$$

$$\forall \Box \exists \Diamond a \longleftarrow \text{alternative (direct) proof}$$

have no equivalent **LTL** formula

*Proof* uses the fact that for each **CTL** formula  $\Phi$ :

- either there is **no** equivalent **LTL** formula
- or  $\Phi \equiv \varphi$  where  $\varphi$  is the **LTL** formula obtained from  $\Phi$  by removing of all path quantifiers



There is no LTL formula equivalent to  $\forall \square \exists \diamond a$  COMPARISON4.2-5D

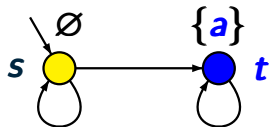
There is no LTL formula equivalent to  $\forall \square \exists \diamond a$  COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

# There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :

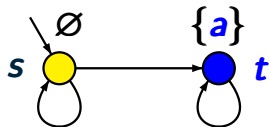


# There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



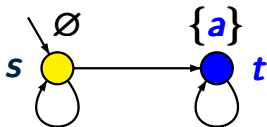
$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

# There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

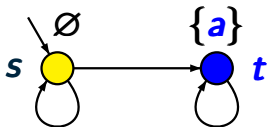
$$\mathcal{T}_1 \models \forall \square \exists \diamond a$$

# There is no LTL formula equivalent to $\forall \square \exists \diamond a$

COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :

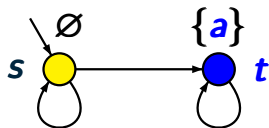


$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

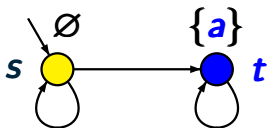
$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :



suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :

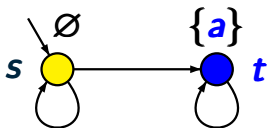


$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\}$$



suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :

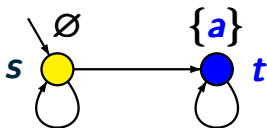


$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1)$$

# There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

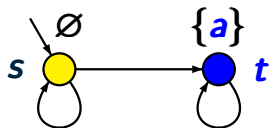
consider the following TS  $\mathcal{T}_2$ :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :



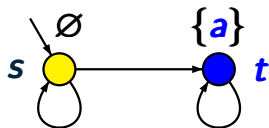
$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

# There is no LTL formula equivalent to $\forall \square \exists \diamond a$ COMPARISON4.2-5D

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :



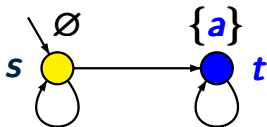
$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

$$\implies \mathcal{T}_2 \models \forall \square \exists \diamond a$$

suppose  $\varphi$  is an **LTL** formula s.t.  $\varphi \equiv \forall \square \exists \diamond a$

consider the following TS  $\mathcal{T}_1$ :



$$\text{Sat}(\exists \diamond a) = \{s, t\}$$

$$\mathcal{T}_1 \models \forall \square \exists \diamond a \implies \mathcal{T}_1 \models \varphi$$

consider the following TS  $\mathcal{T}_2$ :



$$\text{Traces}(\mathcal{T}_2) = \{\emptyset^\omega\} \subseteq \text{Traces}(\mathcal{T}_1) \subseteq \text{Words}(\varphi)$$

$$\text{Hence: } \mathcal{T}_2 \models \varphi$$

$$\implies \mathcal{T}_2 \models \forall \square \exists \diamond a \quad \text{contradiction !!}$$

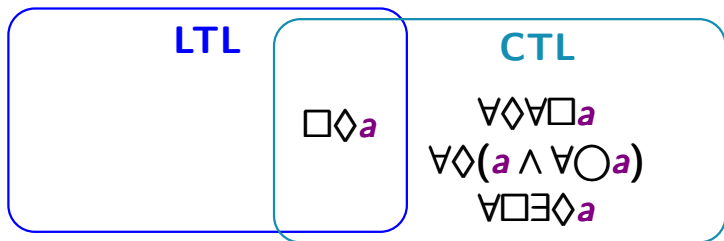
# Expressiveness of LTL and CTL

COMPARISON4.2-5E

The expressive powers of **LTL** and **CTL** are incomparable

The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent **LTL** formula

The **LTL** formula  $\Diamond\Box a$  has no equivalent **CTL** formula

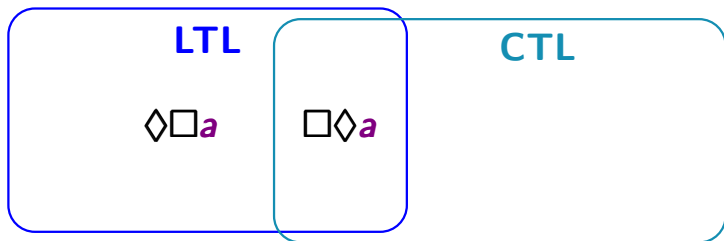


# Expressiveness of LTL and CTL

The expressive powers of **LTL** and **CTL** are incomparable

The **CTL** formulas  $\forall\Diamond(a \wedge \forall\bigcirc a)$ ,  $\forall\Diamond\forall\Box a$  and  $\forall\Box\exists\Diamond a$  have no equivalent **LTL** formula

The **LTL** formula  $\Diamond\Box a$  has no equivalent **CTL** formula







There is no **CTL** formula which is equivalent to the **LTL** formula  $\diamond\Box a$

There is no **CTL** formula which is equivalent to the **LTL** formula  $\diamond\Box a$

*Proof (sketch):* provide sequences  $(\mathcal{T}_n)_{n \geq 0}$ ,  $(\mathcal{T}'_n)_{n \geq 0}$  of transition systems such that for all  $n \geq 0$ :

- (1)  $\mathcal{T}_n \not\models \diamond\Box a$
- (2)  $\mathcal{T}'_n \models \diamond\Box a$

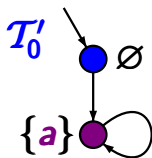
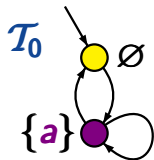
There is no **CTL** formula which is equivalent to the **LTL** formula  $\diamond\Box a$

*Proof (sketch):* provide sequences  $(\mathcal{T}_n)_{n \geq 0}$ ,  $(\mathcal{T}'_n)_{n \geq 0}$  of transition systems such that for all  $n \geq 0$ :

- (1)  $\mathcal{T}_n \not\models \diamond\Box a$
- (2)  $\mathcal{T}'_n \models \diamond\Box a$
- (3)  $\mathcal{T}_n$  and  $\mathcal{T}'_n$  satisfy the same **CTL** formulas length  $\leq n$

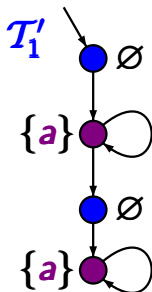
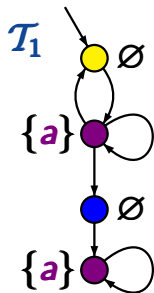
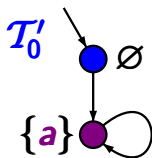
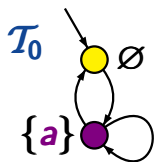
# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6



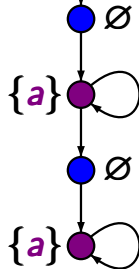
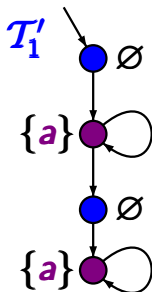
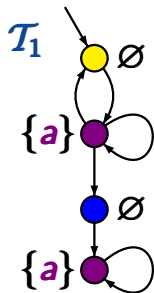
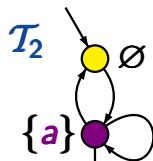
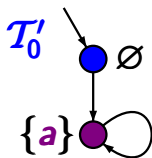
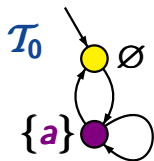
# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6



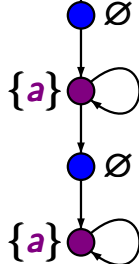
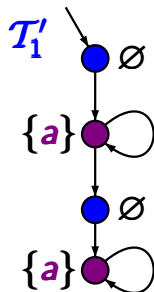
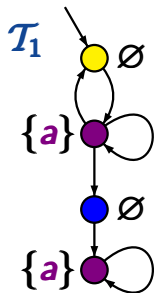
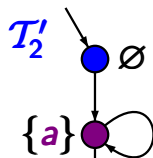
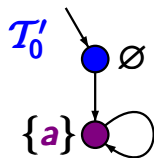
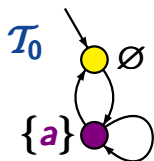
# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6



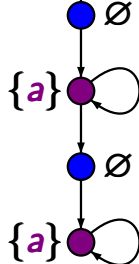
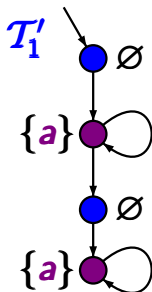
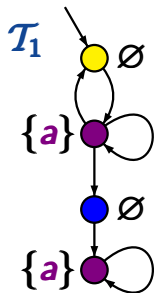
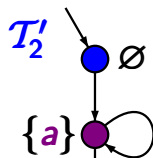
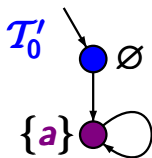
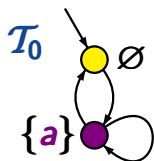
# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6



# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6

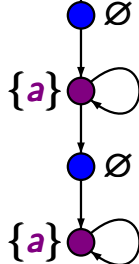
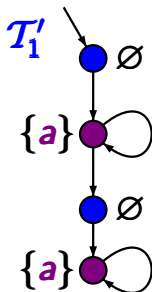
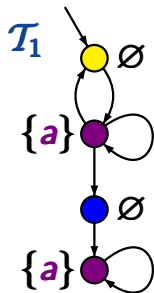
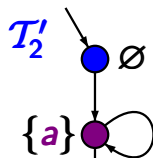
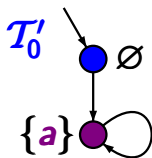
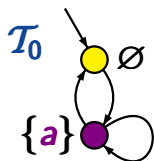


$\mathcal{T}_n \not\models \diamond \square a$



# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-6

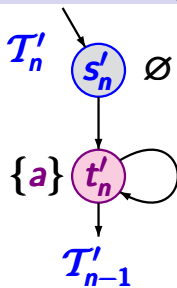
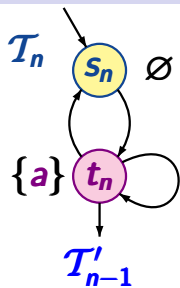


$\mathcal{T}_n \not\models \diamond \Box a$

$\mathcal{T}'_n \models \diamond \Box a$

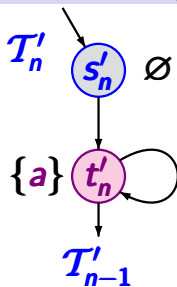
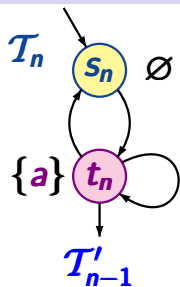
# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-7



# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-7

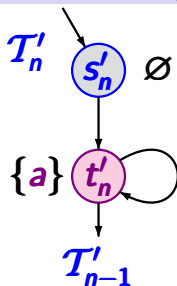
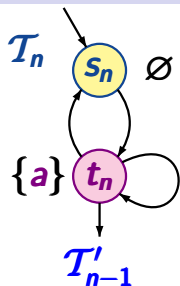


$$\mathcal{T}_n \not\models \diamond \Box a$$

$$\mathcal{T}'_n \models \diamond \Box a$$

# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-7



$$\mathcal{T}_n \not\models \diamond \Box a$$

$$\mathcal{T}'_n \models \diamond \Box a$$

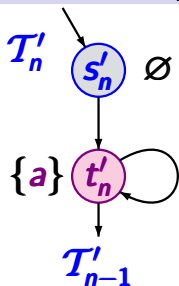
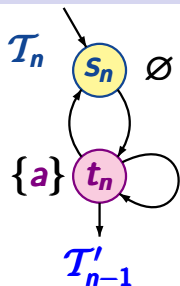
For all **CTL** formulas  $\Phi$  of length  $|\Phi| \leq n$ :

$$s_n \models \Phi \quad \text{iff} \quad s'_n \models \Phi$$

$$t_n \models \Phi \quad \text{iff} \quad t'_n \models \Phi$$

# Transition systems $\mathcal{T}_n$ and $\mathcal{T}'_n$

COMPARISON4.2-7



$$\mathcal{T}_n \not\models \diamond \square a$$

$$\mathcal{T}'_n \models \diamond \square a$$

For all **CTL** formulas  $\Phi$  of length  $|\Phi| \leq n$ :

$$s_n \models \Phi \quad \text{iff} \quad s'_n \models \Phi$$

$$t_n \models \Phi \quad \text{iff} \quad t'_n \models \Phi$$

Hence:  $\mathcal{T}_n$  and  $\mathcal{T}'_n$  fulfill the same **CTL** formulas of length  $\leq n$

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

answer: **no.**

Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

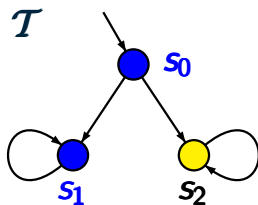
answer: **no.**





Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

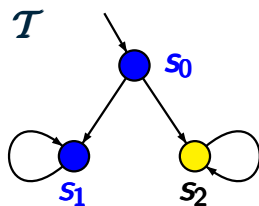
answer: **no.**



$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

answer: **no.**



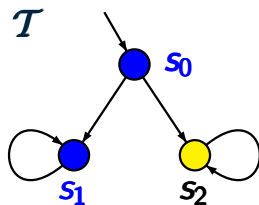
$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

note:  $\pi = s_0 s_2 s_2 s_2 \dots$  is a path in  $\mathcal{T}$  with

$trace(\pi) = \{a\} \emptyset \emptyset \emptyset \dots \notin Words(\Diamond(a \wedge \bigcirc a))$

Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

answer: **no.**

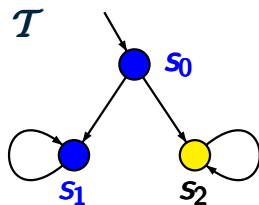


$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \Diamond(a \wedge \exists \bigcirc a)$$

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

answer: **no.**



$$\mathcal{T} \not\models \diamond (a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \diamond (a \wedge \exists \bigcirc a)$$

$$\text{Sat}(\exists \bigcirc a) = \{s_0, s_1\}$$

$$\text{Sat}(\forall \diamond (a \wedge \exists \bigcirc a)) = \{s_0, s_1\}$$

For each **NBA**  $\mathcal{A}$  there is a **CTL** formula  $\Phi$   
such that for all transition systems  $\mathcal{T}$ :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

For each **NBA**  $\mathcal{A}$  there is a **CTL** formula  $\Phi$   
such that for all transition systems  $\mathcal{T}$ :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

**wrong.**

For each **NBA**  $\mathcal{A}$  there is a **CTL** formula  $\Phi$  such that for all transition systems  $\mathcal{T}$ :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

**wrong.** consider, e.g., an NBA  $\mathcal{A}$  with

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\diamond \square a)$$

For each **NBA**  $\mathcal{A}$  there is a **CTL** formula  $\Phi$  such that for all transition systems  $\mathcal{T}$ :

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \text{Traces}(\mathcal{T}) \subseteq \mathcal{L}_\omega(\mathcal{A})$$

**wrong.** consider, e.g., an NBA  $\mathcal{A}$  with

$$\mathcal{L}_\omega(\mathcal{A}) = \text{Words}(\diamond \square a)$$

But there is no CTL formula  $\Phi$  such that  $\Phi \equiv \diamond \square a$



# Correct or wrong?

If  $\phi$  is **CTL** formula and  $\psi$  an **LTL** formula such that  $\phi \equiv \psi$  then  $\neg\phi \equiv \neg\psi$

# Correct or wrong?

If  $\phi$  is **CTL** formula and  $\psi$  an **LTL** formula such that  $\phi \equiv \psi$  then  $\neg\phi \equiv \neg\psi$

**wrong.**

# Correct or wrong?

If  $\phi$  is **CTL** formula and  $\varphi$  an **LTL** formula such that  $\phi \equiv \varphi$  then  $\neg\phi \equiv \neg\varphi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \varphi = \Box\Diamond a$$

# Correct or wrong?

If  $\phi$  is **CTL** formula and  $\psi$  an **LTL** formula such that  $\phi \equiv \psi$  then  $\neg\phi \equiv \neg\psi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \psi = \Box\Diamond a$$

- $\phi \equiv \psi$

If  $\phi$  is **CTL** formula and  $\psi$  an **LTL** formula such that  $\phi \equiv \psi$  then  $\neg\phi \equiv \neg\psi$

wrong. E.g.,

$$\phi = \forall\Box\forall\Diamond a, \quad \psi = \Box\Diamond a$$

- $\phi \equiv \psi$
- there is no CTL formula that is equivalent to

$$\neg\psi \equiv \Diamond\Box\neg a$$

# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

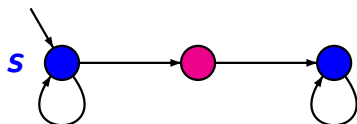
**wrong.**

# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in Paths(s)$  with  
 $\pi \models \square \diamond a$

wrong.



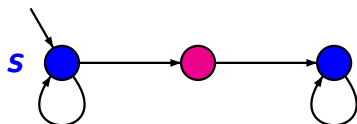


# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  $\pi \models \square \diamond a$

wrong.

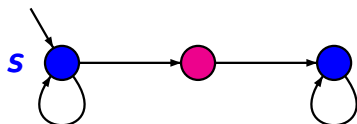


$s \models \exists \square \exists \diamond a$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  $\pi \models \square \diamond a$

wrong.



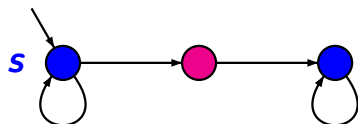
$s \models \exists \square \exists \diamond a$

note that:  $s \models \exists \diamond a$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.



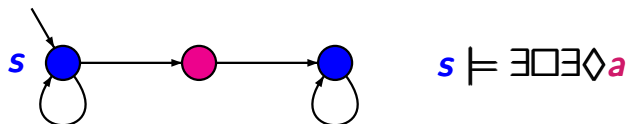
$s \models \exists \square \exists \diamond a$

note that:  $s \models \exists \diamond a$

thus:  $s s s \dots \models \square \exists \diamond a$

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.



note that:  $s \models \exists \diamond a$

thus:  $s s s \dots \models \square \exists \diamond a$

but there is no path where  $\square \diamond a$  holds

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a$$



# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a \text{ iff } s \not\models \forall \square \forall \diamond \neg a$$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$$

$$s \models \exists \diamond \exists \square a \text{ iff } s \not\models \forall \square \forall \diamond \neg a$$

$$\text{iff } s \not\models \square \diamond \neg a$$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$$\begin{aligned} \exists \diamond \exists \square a &\equiv \neg \forall \square \forall \diamond \neg a \\ s \models \exists \diamond \exists \square a &\text{ iff } s \not\models \forall \square \forall \diamond \neg a \\ &\text{ iff } s \not\models \square \diamond \neg a \equiv \neg \diamond \square a \end{aligned}$$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

$s \models \exists \diamond \exists \square a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \diamond \square a$

correct.

$\exists \diamond \exists \square a \equiv \neg \forall \square \forall \diamond \neg a$   
 $s \models \exists \diamond \exists \square a$  iff  $s \not\models \forall \square \forall \diamond \neg a$   
iff  $s \not\models \square \diamond \neg a \equiv \neg \diamond \square a$   
iff there is a path  $\pi \dots$

# Correct or wrong?

COMPARISON4.2-11

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

## Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct**

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$



# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

**correct**

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

correct  $\mathcal{T} \not\models \neg\exists\Box a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

correct as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

correct  $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state  $s$  with  $s \models \neg\exists\Box a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

**correct**  $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state  $s$  with  $s \not\models \neg\exists\Box a$

iff there is an initial state  $s$  with  $s \models \exists\Box a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\Box a$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \Box a$

**correct**  $\mathcal{T} \not\models \neg\exists\Box a$

iff there is an initial state  $s$  with  $s \not\models \neg\exists\Box a$

iff there is an initial state  $s$  with  $s \models \exists\Box a$

iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  $\pi \models \Box a$

# Correct or wrong?

There is an **LTL** formula  $\varphi$  with  $\varphi \equiv \neg\exists\Diamond\exists\Box a$

**correct** as  $\neg\exists\Diamond\exists\Box a \equiv \forall\Box\forall\Diamond\neg a \equiv \Box\Diamond\neg a$

$\mathcal{T} \not\models \neg\exists\varphi$  iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  
 $\pi \models \varphi$

**correct**  $\mathcal{T} \not\models \neg\exists\varphi$

iff there is an initial state  $s$  with  $s \not\models \neg\exists\varphi$

iff there is an initial state  $s$  with  $s \models \exists\varphi$

iff there is a path  $\pi \in \text{Paths}(\mathcal{T})$  with  $\pi \models \varphi$

# Correct or wrong?

COMPARISON4.2-11A

$\mathcal{T} \not\models \neg \forall \Box a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \Box a$



# Correct or wrong?

$\mathcal{T} \not\models \neg \forall \Box a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \Box a$

wrong.

# Correct or wrong?

$\mathcal{T} \not\models \neg \forall \Box a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \Box a$

wrong.

$\mathcal{T} \not\models \neg \forall \Box a$

# Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state  $s$  with  $s \not\models \neg \forall \square a$

# Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state  $s$  with  $s \not\models \neg \forall \square a$

iff there is an initial state  $s$  with  $s \models \forall \square a$

# Correct or wrong?

$\mathcal{T} \not\models \neg \forall \square a$  iff for all paths  $\pi \in \text{Paths}(\mathcal{T})$ :  
 $\pi \models \square a$

wrong.

$\mathcal{T} \not\models \neg \forall \square a$

iff there is an initial state  $s$  with  $s \not\models \neg \forall \square a$

iff there is an initial state  $s$  with  $s \models \forall \square a$

but there might be another initial state  $t$   
s.t.  $t \not\models \forall \square a$

# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

# Correct or wrong?

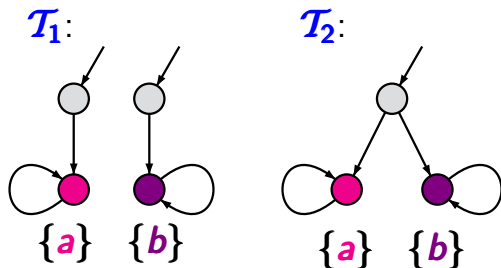
If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

wrong.

# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

wrong.

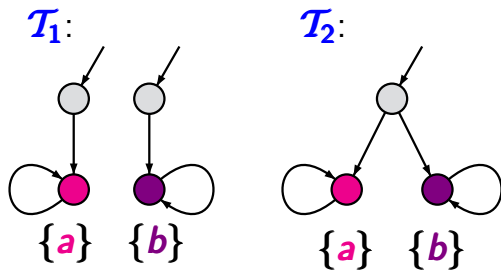




# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

wrong.

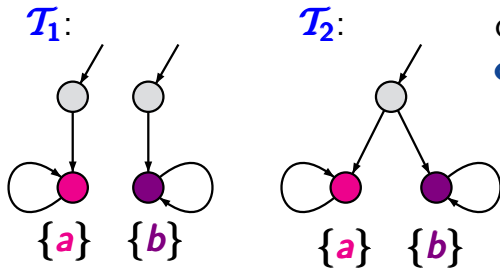


$\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent

# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

wrong.



consider the CTL formula  
 $\phi = \exists \bigcirc a \wedge \exists \bigcirc b$

$$\mathcal{T}_1 \not\models \phi$$

$$\mathcal{T}_2 \models \phi$$

$\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent