Exercise sheet 3 on Discrete Mathematics

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Exercise 3.1. *Prove Lemma 3.4.3: a Hintikka set Γ is consistent, and moreover, for each formula φ, either $\varphi \notin \Gamma$, or $\neg\varphi \notin \Gamma$.*

Exercise 3.2. *Given a formula φ, let $\mathfrak{H}(\varphi)$ be the set of Hintikka sets containing φ, that is, $\mathfrak{H}(\varphi) = \{\, \Gamma \subseteq FOF \mid \varphi \in \Gamma$ and Γ is a Hintikka set $\}$. We say that $\Gamma \in \mathfrak{H}(\varphi)$ is minimal if, for each $\Gamma' \in \mathfrak{H}(\varphi)$, it holds that $\Gamma' \subseteq \Gamma$ implies $\Gamma' = \Gamma$; we denote by $\mathfrak{m}(\varphi)$ the set of minimal Hintikka sets in $\mathfrak{H}(\varphi)$, that is, $\mathfrak{m}(\varphi) = \{\, \Gamma \in \mathfrak{H}(\varphi) \mid \Gamma$ is minimal $\}$.*

1. *Provide a minimal Hintikka set $\Gamma_\varphi \in \mathfrak{m}(\varphi)$ for the formula*

$$\varphi = \forall x \forall y (\neg(x \approx y) \to (R(x,y) \to \neg R(y,x)))$$

   under the assumption that $VS = \{x, y\}$, $FS = CS = \{a, b\}$, $PS = \{R\}$, and $ES = \{\approx\}$.

2. *Prove that $\mathfrak{H}(\varphi) \cap \mathfrak{H}(\neg\varphi) = \emptyset$ for each $\varphi \in FOF$.*

3. *Let $PL \subseteq FOF$ be the set of FOL formulas in which each predicate appears at most once and in which no quantifier $Q \in \{\forall, \exists\}$ occurs. Define a function $c\colon PL \to \mathbb{N}$ such that, for each $\varphi \in PL$, returns the number of different minimal Hintikka sets containing φ.*

Exercise 3.3. *Given an array* `array`*, we say that* `array` *is palindrome if the sequence of expressions obtained by cycling on it from 0 to* **length**(`array`) $- 1$ *is the same as the sequence of expressions obtained by cycling on it from* **length**(`array`) $- 1$ *to 0. For example, the array $[0, 4, 3, 4, 0]$ is palindrome while the array $[0, 4, 3, 4, 1]$ is not.*

Write a well-formed **PROC** *program that returns 1 or 0 depending on whether the provided* `array` *is palindrome or not, respectively.*

Exercise 3.4.

1. *Write a well-formed* **PROC** *program* `maxFactor` *that, on input a number* `n` > 1*, returns the greatest factor f of* `n` *such that $f \neq$* `n`*. (Remember that the expression Exp/Exp corresponds to the integer division, e.g., $5/2 = 2$.)*

2. *How would* `maxFactor` *be changed if the expression Exp/Exp would correspond to the rational division (e.g., $5/2 = 2.5$)?*

Exercise 3.5. *The knapsack problem is a well-known problem in combinatorial optimization; its statement is as follows: given a set of items, each with a weight and a value, choose some of the items so that the total weight is at most the knapsack capacity and the total value is as large as possible.*

For this exercise, consider a simplified version of the problem where all items have the same value.

Provide a well-formed **PROC** *greedy program* `knapsack` *that, on input the knapsack capacity* `c` *and the array of weights* `w`*, returns a vector* `s` *with* $\mathbf{length}(\mathtt{s}) = \mathbf{length}(\mathtt{w})$ *so that* $\mathtt{s}[i] = 1$ *if and only if the item* i *has been choosen for being put in the knapsack.*

Exercise 3.6. *For the following statements, decide whether they are correct; if yes, prove them, otherwise provide a counterexample.*

1. *If* $f(x) \in \Theta(g(x))$ *and* $g(x) \in \Theta(h(x))$*, then* $f(x) \in \Theta(h(x))$*.*

2. *Given two functions* $f(n)$ *and* $g(n)$*, it is true that* $f(n) \in \Theta(g(n))$ *if and only if* $f(n) \in O(g(n))$ *and* $g(n) \in O(f(n))$*.*

3. *Given two functions* $f_1, f_2 \colon \mathbb{N} \to \mathbb{R}$ *so that* $f_1(n) \in \Theta(g(n))$ *and* $f_2(n) \in \Theta(g(n))$ *for some function* g*, it is true that* $(f_1 - f_2)(n) \in \Theta(g(n))$*.*